# ECE 641
# Advanced Topics in Supervisory Control for Discrete Event Systems
Lecture 9

Associate Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineering – Çankaya University

PhD Course in Electronic and Communication Engineering
Credits (3/0/3)
Course webpage: http://ece641.cankaya.edu.tr/





## Strongly Connected Components: Subautomaton

**Definition (Subautomaton)**

Let $G = (X, \Sigma, \delta, x_0, X_{\mathrm{m}})$ and $G' = (X', \Sigma, \delta', x_0', X_{\mathrm{m}}')$ be finite state automata. $G'$ is a *subautomaton* of $G$, denoted as $G' \sqsubseteq G$ if either $G'$ is the empty automaton ($X' = \emptyset$), or $X' \subseteq X$, and for all $x \in X'$ and $\sigma \in \Sigma$, it holds that $\delta'(x, \sigma)! \Rightarrow \delta'(x, \sigma) = \delta(x, \sigma)$. $G'$ is a *strict subautomaton* of $G$ if additionally $\delta(x, \sigma) \in X' \Rightarrow \delta'(x, \sigma) = \delta(x, \sigma)$.

**Remarks**

- State set of $G'$ is a subset of state set of $G$
- Any transition of $G'$ is also a transition of $G$
- The subautomaton is strict if any state of $G'$ has all possible transitions at the corresponding state in $G$

# Strongly Connected Components: Subautomaton

## Example

Gap 1

Klaus Schmidt
Department of Electronic and Communication Engineering – Çankaya University



# Strongly Connected Components: Definition

**Definition (SCC)**

Let $G = (X, \Sigma, \delta, x_0, X_\mathrm{m})$ be an automaton. A subautomaton $G'$ of $G$ with the states $X' \subseteq X$ is called a strongly connected component (SCC) of $G$ if for all state pairs $x, x' \in X'$, there is $u, u' \in \Sigma^*$ s.t. $\delta(x, u) = x'$ and $\delta(x', u') = x$ and for all $X'' \supset X'$, $X''$ is not an SCC of $G$.

## Remarks

- In an SCC, it is possible to reach each state of the SCC from any other state of the SCC
- If one state is added to the SCC this property is no longer valid
- If an SCC consists of only one state, it is called a trivial SCC

## Algorithm

Tarjan, R. E. (1972), "Depth-first search and linear graph algorithms", SIAM Journal on Computing 1 (2): 146–160 (Complexity: $O(|X| + |\delta|)$)

Klaus Schmidt
Department of Electronic and Communication Engineering – Çankaya University

# Strongly Connected Components: Definition

**Example**

Gap 2

Klaus Schmidt
Department of Electronic and Communication Engineering – Çankaya University



# State Attraction: State-feedback Supervisor

**Definition (State-feedback Supervisor)**

Assume that $S$ is a supervisor for plant $G$ and the uncontrollable events $\Sigma_u$. $S$ is denoted as a *state-feedback supervisor* for $G$ and $\Sigma_u$ if it can be realized as a subautomaton of $G$, that is, $S \sqsubseteq G$.

**Example**

Gap 3

Klaus Schmidt
Department of Electronic and Communication Engineering – Çankaya University

# State Attraction: Invariant Set

## Definition (Invariant Set)

Consider an automaton $G = (X, \Sigma, \delta, x_0, X_{\mathrm{m}})$ and an uncontrollable event set $\Sigma_{\mathrm{u}}$. We denote a subset $X' \subseteq X$ as an *invariant set* in $G$ if no transition from a state in $X'$ leaves this set, that is,

$$\forall x \in X' \text{ and } \sigma \in \Sigma \text{ it holds that } \delta(x,\sigma)! \Rightarrow \delta(x,\sigma) \in X'$$

**Example**

Gap 4





# State Attraction: Strong Attractor

## Definition (Strong Attractor)

Let $A \subseteq X' \subseteq X$ and assume that $A, X'$ are invariant sets in $G$. Then, $A$ is denoted as a *strong attractor* for $X'$ in $G$ if

- the strict subautomaton of $G$ with the state set $X' \setminus A$ is acyclic
- $\forall x \in X'$, there is $u \in \Sigma^\star$ such that $\delta(x,u) \in A$

Convergence time: longest path from any state $x \in X'$ to the set $A$.

**Remarks**

- There is no non-trivial SCC in $X' \setminus A$
  $\Rightarrow$ It is not possible to generate an arbitrarily long string outside $A$
  $\Rightarrow$ $A$ is reached from any state in $X'$ after a finite number of event occurrences
- Verification complexity: $O(|X| + |\delta|)$

# State Attraction: Strong Attractor

## Example

Gap 5

Klaus Schmidt
Department of Electronic and Communication Engineering – Çankaya University



# State Attraction: Weak Attractor

**Definition (Weak Attractor)**

Let $A \subseteq X' \subseteq X$ and assume that $A, X'$ are invariant sets in $G$. Let $\Sigma_{\mathrm{u}} \subseteq \Sigma$ be a set of uncontrollable events. $A$ is denoted as a *weak attractor* for $X'$ in $G$ if there exists a state-feedback supervisor $S \sqsubseteq G$, such that $A$ is a strong attractor for $X'$ in $S$.

## Example

Gap 6

Klaus Schmidt
Department of Electronic and Communication Engineering – Çankaya University

# State Attraction: Weak Attractor

## Uniqueness

- A state-feedback supervisor for weak attraction is not unique

## Example

Gap 7





# State Attraction: Supremal Subset

**Lemma (Supremal Subset)**

*There is a set $\Omega_G(A) \subseteq X$, that denotes the supremal subset of $X$ such that $A$ is a weak attractor for $\Omega_G(A)$ in $G$. $\Omega_G(A)$ can be computed with complexity $\mathcal{O}(|X| \cdot |\Sigma|)$.*

## Literature

- Introduction of state attraction

Brave Y. and Heymann M. (1990), Stabilization of discrete-event processes, International Journal of Control, 51:1101–1117.

C. M. Özveren, A. S. Willsky, and P. J. Antsaklis. (1991), Stability and stabilizability of discrete event dynamic systems. Journal of the Association of Computing Machinery 38(2), 729–751

- Algorithm

Kumar R., Garg V. and Marcus S. I. (1993), Language stability and stabilizability of discrete event dynamical systems, SIAM Journal of Control and Optimization, 31:132–154.

# State Attraction: Supremal Subset Algorithm

**Input**

- Automaton $G = (X, \Sigma, \delta, x_0, X_{\mathrm{m}})$, invariant set $A$
- Uncontrollable events $\Sigma_{\mathrm{u}}$
- Attractive set $\Omega = A$, waiting set $W = X \setminus A$

**Procedure**

1. Pick state $w \in W$ such that for all $\sigma \in \Sigma$ with $\delta(w, \sigma)!$
   - either $\delta(w, \sigma) \in \Omega$
   - or $\sigma \notin \Sigma_{\mathrm{u}}$
2. Use $\Omega = \Omega \cup \{w\}$ and $W = W \setminus \{w\}$
3. **terminate** if $W = \emptyset$ or there is no more state in $W$ that fulfills 1.
   $\Rightarrow$ The result is $\Omega_G(A) = \Omega$.
4. Otherwise go back to step 1.





# State Attraction: Weak Attractor

**Example**

Gap 8

# State Attraction: Weak Attractor

## Example

Gap 9





# State Attraction: Extensions

## Minimally Restrictive Optimal Supervisor for State Attraction

- $S$ is optimal in the sense that there is no supervisor $S'$ with a faster convergence time.
- $S$ is minimally restrictive in the sense that, for any other supervisor $S'$ with the same convergence time as $S$, it holds that $S' \sqsubseteq S$.

  Brave Y. and Heymann M. (1993), On optimal attraction of discrete-event processes, Information Sciences 67:245–276.

## Language Convergence

- Converge to a specification $K$ after a bounded convergence time.

  Kumar R., Garg V. and Marcus S. I. (1993), Language stability and stabilizability of discrete event dynamical systems, SIAM Journal of Control and Optimization, 31:132–154.

  Willner Y. and Heymann M. (1995), "Language convergence in controlled discrete-event systems,Äutomatic Control, IEEE Transactions on, 40(4):616–627.