ECE 641
Advanced Topics in Supervisory Control for Discrete Event Systems
Lecture 8

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PhD Course in Electronic and Communication Engineering
Credits (3/0/3)
Course webpage: http://ece641.cankaya.edu.tr/

Decentralized Control: Basics

Components

- Plant $G = (X, \Sigma, \delta, x_0, X_\mathrm{m})$
- Specification $K \subseteq L_\mathrm{m}(G)$; automaton $C = (Y, \Sigma, \gamma, y_0, Y_\mathrm{m})$
- Observable event sets $\Sigma_{\mathrm{o},i} \subseteq \Sigma$ for $i = 1, \ldots, n$
  $\Rightarrow \Sigma_\mathrm{o} = \bigcup_{i=1}^{n} \Sigma_{\mathrm{o},i}$
- Controllable events sets $\Sigma_{\mathrm{c},i} \subseteq \Sigma$ for $i = 1, \ldots, n$
  $\Rightarrow \Sigma_\mathrm{c} = \bigcup_{i=1}^{n} \Sigma_{\mathrm{c},i}$
- Supervisors under partial observation S_i, $i = 1, \ldots, n$

Task

- Determine the supervisors S_i, $i = 1, \ldots, n$ such that the closed loop language is K

Decentralized Control: Architecture

Decentralized Supervisory Control Loop

Gap 1

Idea

- Each S_i, $i = 1, \ldots, n$ only observes $\Sigma_{o,i}$
- Each S_i, $i = 1, \ldots, n$ can disable events in $\Sigma_{c,i}$
- Control decision of $S_1, \ldots, S_n$ is fused to get overall control decision

Decentralized Control: Decision Fusion

Conjunctive Architecture

- Intersection of disablement decisions
  $\Rightarrow$ If one S_i disables an event $\sigma \in \Sigma_{c,i}$, it is disabled

Disjunctive Architecture

- Union of enablement decisions
  $\Rightarrow$ If one S_i enables an event $\sigma \in \Sigma_{c,i}$, it is enabled

Alternative Architectures

- See for example

  Chakib, H.; Khoumsi, A., Multi-Decision Supervisory Control: Parallel Decentralized Architectures Cooperating for Controlling Discrete Event Systems, Automatic Control, IEEE Transactions on, vol.56, no.11, pp.2608–2622, 2011.

$\Rightarrow$ We will study the conjunctive architecture
$\Rightarrow$ Closed loop is represented by $(||_{i=1}^{n} S_i)||G$

Decentralized Control: Conjunctive Architecture

Example

Gap 2

Decentralized Control Conjunctive Architecture Decidability Literature

Decentralized Control: Conjunctive Architecture

Example

Gap 3

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Decentralized Control: Conjunctive Architecture

Example

Gap 4

Question

- Under which condition is the conjunctive architecture sufficient for realizing a given specification K?

Conjunctive Architecture: Co-Observability

Definition (Co-observability)

Let G be a plant automaton, $K \subseteq L_{\mathrm{m}}(G)$ be a specification, $\Sigma_{\mathrm{o},i}$ be the sets of observable events and $\Sigma_{\mathrm{c},i}$ be the sets of controllable events for $i = 1, \ldots, n$. Define the natural projections $p_i : \Sigma^\star \to \Sigma_{\mathrm{o},i}^\star$ for $i = 1, \ldots, n$. K is co-observable for G, $\Sigma_{\mathrm{o},i}$, $\Sigma_{\mathrm{c},i}$, $i = 1, \ldots, n$ if for all $s \in \overline{K}$ and all $\sigma \in \Sigma_{\mathrm{c}}$

$$(s\sigma \notin \overline{K}) \text{ and } (s\sigma \in L(G))$$
$$\Rightarrow \exists i \in \{1, \ldots, n\} \text{ such that } p_i^{-1}p_i(s)\sigma \cap \overline{K} = \emptyset \text{ and } \sigma \in \Sigma_{\mathrm{c},i}.$$

Remarks

- Assume that σ has to be disabled after s
- There should be at least one supervisor that is sure that σ must be disabled and that actually can disable σ after observing $p_i(s)$

Conjunctive Architecture: Special Cases

Observability

- Co-observability becomes observability if
  - for some i, $\Sigma_{o,i} = \Sigma$ and $\Sigma_{c,i} = \Sigma_c$
  - for $j \in \{1, \ldots, n\} \setminus \{i\}$, $\Sigma_{o,j} = \emptyset$

Independent Observability

- Co-observability becomes equivalent to observability of K for G and all p_i and $\Sigma_{c,i}$, $i = 1, \ldots, n$ if $\Sigma_{c,i} \cap \Sigma_{c,j} = \emptyset$ for all $i \neq j$

Example

Gap 5

Conjunctive Architecture: Co-observability

Example

Gap 6

Conjunctive Architecture: Main Theorem

Theorem (Decentralized Supervisory Control)

Let G be a plant automaton, $K \subseteq L_{\mathrm{m}}(G)$ be a specification, $\Sigma_{o,i}$ be the sets of observable events and $\Sigma_{c,i}$ be the sets of controllable events for $i = 1, \ldots, n$. Define the natural projections $p_i : \Sigma^\star \to \Sigma_{o,i}^\star$ for $i = 1, \ldots, n$. There is a nonblocking decentralized supervisor S_i for $i = 1, \ldots, n$ in the conjunctive architecture such that

$$L_m(S_1 || \cdots || S_n || G) = K \text{ and } L(S_1 || \cdots || S_n || G) = \overline{K}$$

if and only if

1. *K is controllable for G and Σ_u.*
2. *K is co-observable for G, $\Sigma_{o,i}$ and $\Sigma_{c,i}$ for $i = 1, \ldots, n$.*
3. *K is $L_m(G)$-closed.*

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Decentralized Control | Conjunctive Architecture | Decidability | Literature

Conjunctive Architecture: Supervisor Computation

Decentralized Supervisor S_i

- Supervisor under partial observation for $p_i(K)$

Example

Gap 7

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Conjunctive Architecture: Co-observability Verification

Verifier (for $n = 2$)

- $V = (W, \Gamma, \omega, w_0, W_{\mathrm{m}})$
- $W \subseteq Y \times Y \times Y \times X \cup \{NCO\}$
- $\Gamma = (\Sigma \cup \{\epsilon\}) \times (\Sigma \cup \{\epsilon\}) \times (\Sigma \cup \{\epsilon\}) \times (\Sigma \cup \{\epsilon\})$
- $w_0 = (y_0, y_0, y_0, x_0)$
- $W_{\mathrm{m}} = \{NCO\}$
- Transition relation: analogous rules to observability verification
  - First copy of C follows p_1
  - Second copy of C follows p_2
  - Third copy of C follows real specification
  - Fourth entry follows plant

  $\Rightarrow$ Co-observability holds if $L_{\mathrm{m}}(V) = \emptyset$
- Complexity: $O(|Y|^3|X|)$

Decidability: Decentralized Supervisory Control

Undecidability Result

- Assume G, $K \subseteq L_{\mathrm{m}}(G)$, $\Sigma_{\mathrm{o},i}$ and $\Sigma_{\mathrm{c},i}$ for $i = 1, \ldots, n$
- Assume that K is not co-observable
- The problem of finding a decentralized supervisor $S_1, \ldots, S_n$ such that $L_{\mathrm{m}}(S_1|| \cdots S_n||G) \subseteq K$ is undecidable

  S. Tripakis, Undecidable problems of decentralized observation and control, Information Processing Letters, vol. 90, no. 1, pp. 21–28, 2004.

Further Literature

- Rudie, K.; Willems, J.C., The computational complexity of decentralized discrete-event control problems, Automatic Control, IEEE Transactions on, vol.40, no.7, pp.1313–1319, 1995.
- Yoo, T.-S., and S. Lafortune, A general architecture for decentralized supervisory control of discrete-event systems, Discrete Event Dynamic Systems: Theory & Applications, Vol. 12, No. 3, pp. 335–377, 2002.