ECE 641
Advanced Topics in Supervisory Control for Discrete Event Systems

Lecture 6

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PhD Course in Electronic and Communication Engineering
Credits (3/0/3)
Course webpage: http://ece641.cankaya.edu.tr/

Supervision Under Partial Observation: Basics

Components

- Plant automaton $G = (X, \Sigma, \delta, x_0, X_\mathrm{m})$
- Specification $K \subseteq L_\mathrm{m}(G)$, specification automaton $C = (Y, \Sigma, \gamma, y_0, Y_\mathrm{m})$
- Controllable events Σ_c, uncontrollable events Σ_u
- Observable events Σ_o, unobservable events Σ_uo
- Supervisor under partial observation: $S = (Q, \Sigma, \nu, q_0, Q_\mathrm{m})$ such that
  - $L(S)$ is controllable for G and Σ_u
  - for all $s, s' \in L(S)$ such that $p(s) = p(s')$ and all $\sigma \in \Sigma_\mathrm{c}$ it must hold that $s\sigma \in L(S) \Leftrightarrow s'\sigma \in l(S)$

Remarks

- A supervisor under partial observation only sees observable events
  $\Rightarrow$ Strings with same observation must lead to same control decision

Supervision Under Partial Observation: Illustration

Examples

Gap 1

Supervision Under Partial Observation Observability

Supervision Under Partial Observation: Illustration

Examples

Gap 2

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Supervision Under Partial Observation: Questions

Observations

- Strings with the same observation require the same control action
- It is possible to disable controllable unobservable events even though they cannot be observed

Existence

- Determine under which conditions a supervisor under partial observation exists such that $L_{\mathrm{m}}(G||S) = K$ and $L(S||G) = \overline{K}$

Synthesis

- Determine a (largest possible) sublanguage $K_{sub} \subseteq K$ such that $L_{\mathrm{m}}(G||S) = K_{sub}$ and $L(S||G) = \overline{K}_{sub}$

Reminder: $L_{\mathrm{m}}(G)$-Closure

- $K \subseteq L_{\mathrm{m}}(G)$ is $L_{\mathrm{m}}(G)$-closed if $K = \overline{K} \cap L_{\mathrm{m}}(G)$

Observability: Definition

Definition (Observability)

Let G be a plant automaton, $K \subseteq L_{\mathrm{m}}(G)$ a specification language, Σ_c a controllable event set, Σ_{o} an observable event set and $p : \Sigma^\star \to \Sigma_{\mathrm{o}}^\star$ the associated natural projection. K is denoted as *observable* for G, Σ_{c} and Σ_{o} if for all $s, s' \in \overline{K}$ with $p(s) = p(s')$ and for all $\sigma \in \Sigma_{\mathrm{c}}$

$$s\sigma \in L(G) \text{ and } s'\sigma \in \overline{K} \Rightarrow s\sigma \in \overline{K}.$$

Remarks

- Consider strings s, s' with same observation
- σ is possible after s in the plant and σ is allowed after s' by the specification

$\Rightarrow$ σ should also be allowed by the specification after s (otherwise, supervisor has to both enable and disable σ after observing $p(s) = p(s')$)

Observability: Illustration

Examples

Gap 3

Observability: Supervisor Existence

Theorem (Observability Theorem)

Let G be a plant automaton, $K \subseteq L_{\mathrm{m}}(G)$ a specification language, Σ_c a controllable event set, Σ_{o} an observable event set and $p : \Sigma^\star \to \Sigma_{\mathrm{o}}^\star$ the associated natural projection. There exists a nonblocking supervisor under partial observation S such that $L_{\mathrm{m}}(G||S) = K$ and $L(G||S) = \overline{K}$ if and only if

- *K is controllable for G and Σ_{u}*
- *K is observable for G, Σ_{c} and Σ_{o}*
- *K is $L_{\mathrm{m}}(G)$-closed*

Remarks

- The theorem assumes that the supervisor is non-marking
- We cannot use the specification as supervisor (unobservable events)

Observability: Supervisor Computation

Procedure

- Compute $S = (Q, \Sigma, \nu, q_0, Q_\mathrm{m})$ as observer automaton of C
  - $q_0 = UR(y_0)$
  - Transitions from any state $q \in Q$ with observable event $\sigma \in \Sigma_\mathrm{o}$: $\mu(q, \sigma) = OR(q, \sigma)$
- For each state $q \in Q$ and each $y \in q$: add unobservable events that are possible in C at y as selfloops to S:

$$(\forall q \in Q)(\forall y \in q)(\forall \sigma \in \Sigma_\mathrm{uo})\, \gamma(y, \sigma)! \Rightarrow \nu(q, \sigma) = q$$

Remarks

- S changes its state only with observable transitions
- Whenever an unobservable event is allowed by the specification, a selfloop transition is introduced in S
  $\Rightarrow$ Strings with the same observation lead to the same control action

Observability: Supervisor Computation

Example

Gap 4

Observability: Verification

Input

- G, C, Σ_{c}, Σ_{o}

Result

- Verifier automaton $V = (W, \Gamma, \omega, w_0, W_{\mathrm{m}})$
- $W \subseteq Y \times Y \times X \cup \{NO\}$
- $\Gamma = (\Sigma \cup \{\epsilon\}) \times (\Sigma \cup \{\epsilon\}) \times (\Sigma \cup \{\epsilon\})$
- $w_0 = (y_0, y_0, x_0)$
- $W_{\mathrm{m}} = \{NO\}$
- For all $\sigma \in \Sigma_{\mathrm{o}}$

$$\omega((y_1, y_2, x), (\sigma, \sigma, \sigma)) = (\gamma(y_1, \sigma), \gamma(y_2, \sigma), \delta(x, \sigma))$$

if $\gamma(y_1, \sigma)!$ and $\gamma(y_2, \sigma)!$ and $\delta(x, \sigma)!$

Observability: Verification

Result

- For all $\sigma \in \Sigma_{\mathrm{uo}}$

$$\omega((y_1, y_2, x), (\sigma, \epsilon, \epsilon)) = (\gamma(y_1, \sigma), y_2, x)$$

if $\gamma(y_1, \sigma)!$

$$\omega((y_1, y_2, x), (\epsilon, \sigma, \sigma)) = (y_1, \gamma(y_2, \sigma), \delta(x, \sigma))$$

if $\gamma(y_2, \sigma)!$ and $\delta(x, \sigma)!$

$$\omega((y_1, y_2, x), (\sigma, \epsilon, \sigma)) = NO$$

if $\sigma \in \Sigma_{\mathrm{c}}$, $\gamma(y_1, \sigma)!$, $\neg\gamma(y_2, \sigma)!$ and $\delta(x, \sigma)!$

Observability: Verification

Example

Gap 5

Observability: Verification

Example

Gap 6

Observability: Verification

Properties

- Consider any string (s_1, s_2, s_3) from w_0 to state (y_1, y_2, x) in V
  $\gamma(y_0, s_1) = y_1$, $\gamma(y_0, s_2) = y_2$, $\delta(x_0, s_3) = x$
  $s_i \in \overline{K}$ for $i = 1, 2, 3$
  $s_2 = s_3$ according to the contruction rules
  $p(s_1) = p(s_2) = p(s_3)$
  $\Rightarrow$ State NO is reachable if and only if observability is violated!

Theorem (Observability Verification)

K is observable for G, Σ_c and Σ_o if and only if $L_\mathrm{m}(V) = \emptyset$.

Remarks

- State space of V is $Y \times Y \times X \cup \{NO\}$
  $\Rightarrow$ Complexity of observability verification is $O(|Y|^2\,|X|)$

Observability: Main References

Books

- W.M. Wonham, Lecture notes on control of discrete-event systems, 2011 edition, U. Toronto, Toronto, Canada.
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Papers

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