ECE 641
Advanced Topics in Supervisory Control for Discrete Event Systems

Lecture 11

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PhD Course in Electronic and Communication Engineering
Credits (3/0/3)
Course webpage: http://ece641.cankaya.edu.tr/

Petri Net Languages: Non-Regular Languages

Non-regular Petri Net Languages

- Petri Nets can generate non-regular languages
  ⇒ Petri Nets generate context-sensitive languages

Example

Gap 1

Petri Net Properties: Reachability

Definition (Reachable States)

Let (N, m) be a marked Petri Net with $N = (P, T, A, W)$ and $|P| = n$. The set of reachable states of (N, m) is given as

$$R(N, m) = \{y \in \mathbb{N}^n | \exists s \in T^\star \text{ such that } s \text{ is enabled from } m \text{ and reaches } y\}$$

Remark

- $R(N, m)$ is the set of markings that are reachable from the initial marking along paths with enabled transitions

Reachability Tree

- Root is the initial marking m
- Tree of reachable markings from the initial marking
- Stops if already discovered marking is reached again
  $\Rightarrow$ Reachability tree shows all reachable markings

Petri Net Properties: Reachability

Example

Gap 2

Petri Net Properties: Boundedness

Definition (Boundedness)

Let (N, m) be a marked Petri Net with $N = (P, T, A, W)$. A place $p \in P$ of (N, m) is k-bounded if it holds for all $m' \in R(N, m)$ that $m'[p] \leq k$. p is safe if it is 1-bounded.

Remarks

- p is k-bounded if it does not have more than k tokens for any reachable marking
- p is safe if there is at most one token in p for any reachable marking

Gap 3

Petri Net Properties: Boundedness

Example

Gap 4

Petri Net Properties: Liveness

Definition (Liveness)

Let (N, m) be a marked Petri Net with $N = (P, T, A, W)$. A transition $t \in T$ of (N, m) is live if it possible to reach a marking from where t can be fired from any marking in $R(N, m)$. (N, m) is live if all transitions in T are live.

Remarks

- t is live if it is possible to fire t again from any reachable marking

Gap 5

Petri Net Properties: Deadlock

Definition (Deadlock)

Let (N, m) be a marked Petri Net with $N = (P, T, A, W)$. A reachable marking m' is a deadlock if no transition can fire from m'.

Remarks

- If there is a deadlock, then no transition is live!

Gap 6

Petri Net Properties: Siphons and Traps

Definition (Siphon)

Let (N, m) be a marked Petri Net with $N = (P, T, A, W)$. A set of places $S \subseteq P$ is a siphon if ${}^\bullet S \subseteq S^\bullet$.

Definition (Trap)

Let (N, m) be a marked Petri Net with $N = (P, T, A, W)$. A set of places $S \subseteq P$ is a trap if $S^\bullet \subseteq {}^\bullet S$.

Remarks

- Siphon: For each incoming arc, there is an outgoing arc
  $\Rightarrow$ If a siphon has no tokens, no tokens will ever enter the siphon
- Trap: For each outgoing arc, there is an incoming arc
  $\Rightarrow$ If a trap has a token, it will never become empty

Petri Net Properties: Siphons and Traps

Example

Gap 7

Petri Net Properties: Siphons and Deadlocks

Definition (Ordinary Petri Net)

Let (N, m) be a marked Petri Net with $N = (P, T, A, W)$. (N, m) is ordinary if all weights are 1.

Lemma

A deadlocked Petri Net with marking m' has at least one siphon S such that $\forall p \in S$, $\exists t \in p^\bullet$ with $W(p, t) > m'[p]$.

Gap 8

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Supervisory Control: Motivation

Example

Gap 9

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Supervisory Control: Place Invariant

Definition (Ordinary Petri Net)

Let (N, m) be a marked Petri Net with $N = (P, T, A, W)$ and the incidence matrix C. A vector x is called a place invariant if $x\,C = 0$.

Remarks

- For any marking m': $x\,m' = x\,(m + C\,e) = x\,m + x\,C\,e = x\,m$
  $\Rightarrow$ Weighted number of tokens remains constant for place invariant!

Computation

Gap 10

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Supervisory Control: Derivation

Computation

Gap 11

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Supervisory Control: Derivation

Computation

Gap 12

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Supervisory Control: Procedure

Given

- Incidence matrix C, initial marking m
- Generalized mutual exclusion constraint: $x\,m' \leq b$

Control Place

- Introduce new control place p_c
- Initial marking of p_c: $m(p_c) = b - x\,m$
- Modified incidence matrix $C_c = \begin{bmatrix} C \\ d \end{bmatrix}$
- Compute $d = -x\,C$
  $\Rightarrow$ Connection of control place to remaining places

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Supervisory Control: Example

Computation

Gap 13

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