ECE 641
Advanced Topics in Supervisory Control for Discrete Event Systems

Lecture 10

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PhD Course in Electronic and Communication Engineering
Credits (3/0/3)
Course webpage: http://ece641.cankaya.edu.tr/

Petri Nets: Basics

Historical Perspective

- Petri nets were developed in the early 1960s by C.A. Petri in his Ph.D. dissertation

  C.A. Petri. Kommunikation mit Automaten. PhD thesis, Institut für instrumentelle Mathematik, Bonn, 1962.

Usage of Petri Nets

- Modeling concurrent, distributed, asynchronous behavior in a discrete system
- Many families of Petri Nets for managing continuous/hybrid systems
- Timed systems
- High-level information or tasks of a system
- Variable parameter systems

Petri Nets: Definition

Petri Net

- Four tuple $N = (P, T, A, W)$
- P is a finite set of places
- T is a finite set of transitions
- $A \subseteq (P \times T) \cup (T \times P)$ is a finite set of arcs
- $W : A \to \mathbb{N}$ is a weighting function
- $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$

Graphical Representation

- Places: circles
- Transitions: boxes
- Arcs: arrows
  $\Rightarrow$ Bipartite graph

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Petri Nets: Definition

Example

Gap 1

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Petri Nets: Pre- and Post Sets

Pre-Set $^\bullet x$

- Let $x \in P \cup T$. Then $^\bullet x = \{y \in P \cup T | (y, x) \in A\}$
  $\Rightarrow$ All predecessor nodes (places or transitions) of x

Post-Set $x^\bullet$

- Let $x \in P \cup T$. Then $x^\bullet = \{y \in P \cup T | (x, y) \in A\}$
  $\Rightarrow$ All successor nodes (places or transitions) of x

Example

Gap 2

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Petri Nets: Marked Petri Net

Definition

- Pair (N, m)
- $N = (P, T, A, W)$ is a Petri Net
- $m : P \to \mathbb{N}$ is an initial marking
- Notation for $p \in P$: $m[p]$ is marking of place p

Petri Net Dynamics: Transition Firing

- $t \in T$ is enabled at m iff for all $p \in {}^\bullet t : m[p] \geq W(p, t)$
- $t \in T$ can fire at m iff t is enabled at m
- Firing of t leads to the new marking m' for each $p \in P$ with

$$m'[p] = \begin{cases} m[p] - W(p,t) & \text{if } t \in {}^\bullet p \\ m[p] + W(p,t) & \text{if } t \in p^\bullet \\ m[p] & \text{otherwise} \end{cases}$$

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Petri Nets: Marked Petri Net

Example

Gap 3

Petri Nets: Incidence Matrix and State Equation

Pre- and Post Matrices

- $\forall p \in P$ and $t \in T$: if $(p, t) \in A$, then $Pre[p, t] = W(p, t)$, else $Pre[p, t] = 0$
- $\forall p \in P$ and $t \in T$: if $(t, p) \in A$, then $Post[p, t] = W(p, t)$, else $Post[p, t] = 0$

Incidence Matrix

$$C = Post - Pre$$

State Equation

- For any enabled transition $t \in T$, use vector e_t as unit vector with 1 at position of t and 0 otherwise

$$m' = m + C\, e_t$$

- Note: State equation can be evaluated for any vector e_t but dynamics is only defined for enabled transitions!

Petri Nets: Incidence Matrix and State Equation

Example

Gap 4

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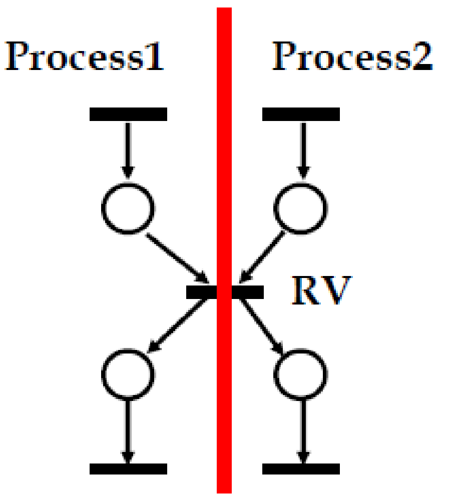
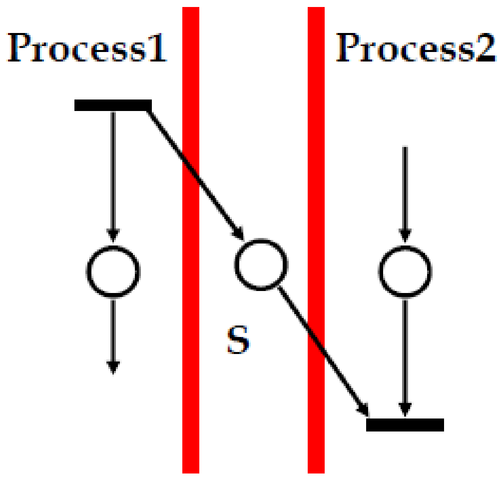
Petri Nets: Structural Elements

Rendezvous



→ Processes meet at common transition RV

Semaphore



→ Continuation of Process 2 depends on completion of some part of process 1

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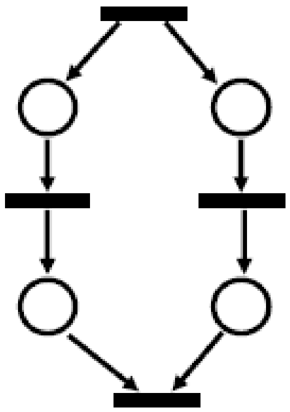
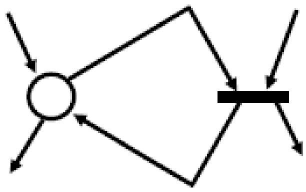
Petri Nets: Structural Elements

Fork Join



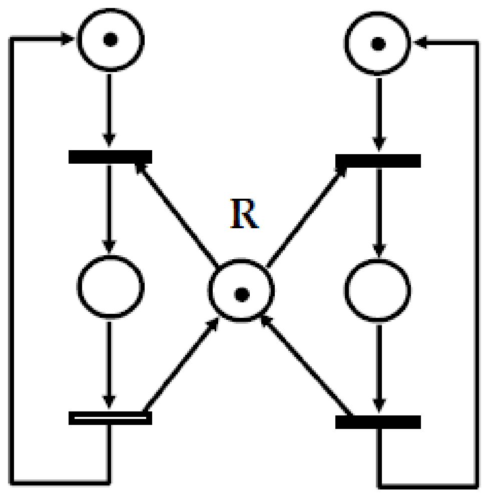
→ Process divides into subprocesses and re-joins

Guard



→ Transition is only possible if there is token in "guard" place

Shared Resource



→ Resource R can only be used by one subprocess at a time

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Petri Nets: Structural Elements

Dining Philosophers Example

- Two philosophers
- Each philosopher either thinks or eats
- There are two forks shared by the philosophers
- Each philosopher can pick any fork if available
- A philosopher thinks if he does not hold any fork
- A philosopher can only eat if he has both forks

Gap 5

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Petri Nets: Structural Elements

Dining Philosophers Petri Net

Gap 6

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Petri Nets PN Languages

Petri Nets: Structural Elements

Dining Philosophers Petri Net

Gap 7

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PN Languages: Definitions

Labeled Petri Net

$$N = (P, T, A, W, E, l, x_0, X_m)$$

- (P, T, A, W) is a Petri net graph
- E is the event set for transition labeling
- $l : T \to E$ is the transition labeling function
- $x_0 \in \mathbb{N}^n$ is the initial state of the net (i.e., the initial number of tokens in each place)
- $X_m \subseteq \mathbb{N}^n$ is the set of marked states of the net.

Generated Language

$$L(N) = \{l(s) \in E^\star | s \in T^\star \text{ and } s \text{ is enabled from } x_0\}$$

Marked Language

$$L_m(N) = \{l(s) \in E^\star | s \in T^\star \text{ and } s \text{ is enabled from } x_0 \text{ and } x_0 + C\,s \in X_m\}$$

PN Languages: Example

Dining Philosophers

Gap 8