

Exercise Sheet 5: Decidability**Problem 11:**

Show the following properties of the computational complexity notions introduced in the lecture:

- a. Reflexivity
- b. Symmetry
- c. Transpose Symmetry

Problem 12:

We consider the abstraction-based supervisory control using the natural observer condition with two plants G_1, G_2 with alphabets Σ_1, Σ_2 and local specification automata C_1, C_2 . Also there is a global specification automaton C over the alphabet $\hat{\Sigma} \subset \Sigma = \Sigma_1 \cup \Sigma_2$. We assume that G_i has n_i states, $i = 1, 2$; C_i has m_i states, $i = 1, 2$ and C has m states.

- a. What is the computational complexity for the monolithic supervisor computation (without abstraction)?
- b. Sketch the architecture for the abstraction-based supervisor computation.
- c. What is the complexity for the abstraction-based computation?

Hint: Assume that the state size of the abstractions \hat{G}_i is \hat{n}_i , $i = 1, 2$.

Use the computational complexity of synchronous composition, SupCon computation, natural observer extension.

- d. Discuss what is the potential advantage of the abstraction-based computation.

Problem 13: [optional]

It is shown in the following paper that finding a minimal alphabet extension for fulfilling the natural observer condition is NP-hard.

L. Feng and W. Wonham, "On the computation of natural observers in discrete-event systems," *Discrete Event Dyn. Syst.: Theor. Appl.*, vol. 20, no. 1, pp. 63–102, Mar. 2010.

Study the proof of this result.