Spring 2013

Exercise Sheet 10/11: Petri Nets

Problem 23:

Consider the Petri net defined by:

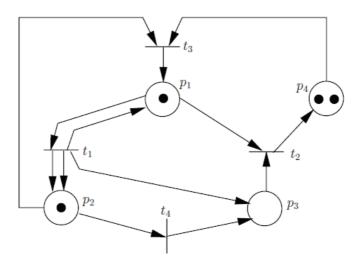
$$\begin{split} P &= \{p1, p2, p3\}; T = \{t1, t2, t3\}; \\ A &= \{(p1, t1), (p1, t3), (p2, t1), (p2, t2), (p3, t3), (t1, p2), (t1, p3), (t2, p3), (t3, p1), (t3, p2)\}. \end{split}$$

Assume that all arc weights are equal to 1 except for W(p1, t1) = 2.

- a. Draw the corresponding Petri Net graph.
- **b.** Let $m = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T$ be the initial marking. Show that transition t1 is not live.
- **c.** Let $m = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix}^T$ be another initial marking. Show that in any subsequent operation of the Petri net, either a deadlock occurs (no transition can be enabled) or a return to m results.

Problem 24:

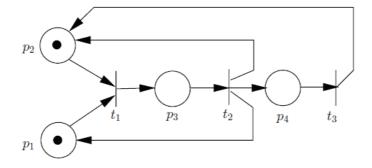
Consider the Petri Net shown in the following figure.



- a. After the Petri net fires twice, find a marking where all transitions are dead.
- **b.** Suppose we want to apply the firing sequence $(t_3, t_1, t_3, t_1, ...)$. Show that this is not possible for all future times.
- **c.** Find the marking m' resulting from the firing sequence (t1, t2, t3, t3, t3).

Problem 25:

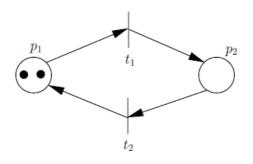
Consider the Petri net in the following figure.



- **a.** Construct its coverability tree and use it to show that the empty state $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ is not reachable. Is this Petri net bounded?
- **b.** Find a trap in the above Petri Net.
- c. Find a siphon in the above Petri Net.

Problem 26:

Consider the Petri net depicted in the following figure. Modify this Petri net in order to get a controlled Petri Net that satisfies the constraint $m'(p2) \leq m'(p1)$ for all reachable markings from the initial marking.



Problem 27:

Consider the Petri Net in the following figure, with the initial marking $m = \begin{bmatrix} k & 0 & k & 0 \end{bmatrix}^T$. Show how to build a controlled version of this net where the following constraint is satisfied in all reachable marking (or explain why this is not possible): $m'(p_3) \ge 3$.

